Comments on "Passive and Impulsive Synchronization of a New Four-Dimensional Chaotic System"[Nonlinear Analysis, doi:10.1016/j.na.2010.09.051]

Emad E. Mahmoud(1,2)*

[1]Department of Mathematics, Faculty of Science, Sohag University, Sohag 82524, Egypt
[2]College of Applied Medical Sciences,Taif University,Turabah, Kingdom of Saudi Arabia
emad_eluan@yahoo.com

Kholod M. Abualnaja

Department of Mathematics, Umm Al-Qura University, P.O. Box 14949, Makkah,Kingdom of Saudi Arabia.

E-mail:kmaboualnaja@uqu.edu.sa

bstract— The aim of this letter is to point out some comments on the article "Nonlinear Analysis 74 (2011) 1146-1154, doi:10.1016/j.na.2010.09.051". We hope to improve the results, published in the[1]. Based on the passivity theory the controllers are designed to synchronize two identical chaotic Qi systems with known parameters. We prove the error system between the two identical chaotic Qi systems with the controller is minimum phase. The analytical results of the controllers, which have been calculated, are tested numerically and good agreement is obtained.

Keywords Passive Control, Minimum Phase, Chaotic

1 Introduction

The main idea of passivity theory is that the passive properties of system can keep the system internally stable. So, to make the system stable, one can design a controller which renders the closed loop system passive using passivity theory[2-5]. Passivity is part of general theory of dissipativity[6, 7]. The passive control has many advantages, e.g., clear physical interpretation, less control effort or ease in implementation[7-10]. Passive systems, like linear circuits containing only positive resistor are stable.

In[1] authors study the synchronization problem for a new chaotic four-dimensional system presented by Qi et al.[11] via passive control and impulsive control methods. But there is major error when the authors of[1] apply the passive control methods to investigate the synchronization of Qi system. Equation (14) in[1]:

$$\begin{aligned}\frac{d}{dt}W(\mathbf{z}) &= \begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix} f(\mathbf{z}),\\ &= (-az_1 + y_2y_3y_4 - x_2x_3x_4)z_1\\ &\quad +(-cz_2 + y_2y_1y_4 - x_2x_1x_4)z_2\\ &\quad +(-dz_3 + y_2y_3y_1 - x_2x_3x_1)z_3.\end{aligned} \tag{1}$$

Why $\frac{d}{dt}W(\mathbf{z}) \le 0$, this is wrong. So, the error system (9) is not minimum phase system and can't be equivalent to a passive system. And passive control method is not suitable to IJSER staff will edit and complete the final formatting of your paper.equation (5) in[1] x must change to $\dot{Z}$ and in equation (9) $\dot{y}_4$ must change to $\dot{e}_4$. The authors employed one control function u in the second equation of response system (8) in[1]. We think one control is not sufficient to achieve synchronization and to proof the error system (9) in[1] is passive system and minimum phase system. In the next section we present our point of view to achieve synchronization of Qi system via passive control. Finally, conclusions are drawn in Section 3.

2 Synchronization of the 4D Chaotic System Via Passive Control

2.1. Formula of the Controller

We employ the same preliminaries and definitions of passivity theory in[1].
The drive dynamical system is described as:

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_2x_3x_4,\\ \dot{x}_2 &= y(x_2 + x_1) - x_1x_3x_4,\\ \dot{x}_3 &= -cx_3 + x_1x_2x_4,\\ \dot{x}_4 &= -dx_4 + x_1x_2x_3.\end{aligned} \tag{2}$$

The response system can be described as:

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + y_2 y_3 y_4 + u_1,\\
\dot{y}_2 &= y(y_2 + y_1) - y_1 y_3 y_4 + u_2,\\
\dot{y}_3 &= -c y_3 + y_1 y_2 y_4 + u_3,\\
\dot{y}_4 &= -d y_4 + y_1 y_2 y_3,
\end{aligned} \qquad (3)$$

The error system is determined as follows:

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + y_2 y_3 e_4 + x_4(y_2 e_3 + x_3 e_2) + u_1,\\
\dot{e}_2 &= b(e_1 + e_2) - y_1 y_3 e_4 - x_4(y_1 e_3 + x_3 e_1) + u_2,\\
\dot{e}_3 &= -c e_3 + y_1 y_2 e_4 + x_4(y_2 e_1 + x_1 e_2) + u_3,\\
\dot{e}_4 &= -d e_4 + y_2 y_3 e_1 + x_1(y_2 e_3 + x_3 e_2),
\end{aligned} \qquad (4)$$

where $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, $e_3 = y_3 - x_3$, $e_4 = y_4 - x_4$. Let $\mathbf{z} = [z_1] = [e_4]$, $\mathbf{w} = [w_1, w_2, w_3]^T = [e_1, e_2, e_3]^T$ where T is transpose thus, system (4) has the form:

$$\begin{aligned}
\dot{z}_1 &= -d z_1 + y_2 y_3 w_1 + x_1(y_2 w_3 + x_3 w_2),\\
\dot{w}_1 &= a(w_2 - w_1) + y_2 y_3 z_1 + x_4(y_2 w_3 + x_3 w_2) + u_1,\\
\dot{w}_2 &= b(w_1 + w_2) - y_1 y_3 z_1 - x_4(y_1 w_3 + x_3 w_1) + u_2,\\
\dot{w}_3 &= -c w_3 + y_1 y_2 z_1 + x_4(y_2 w_1 + x_1 w_2) + u_3,
\end{aligned} \qquad (5)$$

System (5) can be characterized as:

$$\begin{cases} \dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}) + \mathbf{g}(\mathbf{z}, \mathbf{w})\mathbf{w},\\ \dot{\mathbf{w}} = \boldsymbol{l}(\mathbf{z}, \boldsymbol{w}) + \boldsymbol{k}(\mathbf{z}, \mathbf{w})\mathbf{u}, \end{cases} \qquad (6)$$

where:

$$\mathbf{f}(\mathbf{z}) = [-d z_1],\ \mathbf{g}(\mathbf{z}, \mathbf{w}) = [y_2 y_3,\ x_1 x_3,\ x_1 y_2],\ \mathbf{u} = [u_1, u_2, u_3]^T$$

$$\mathbf{l}(\mathbf{z}, \mathbf{w}) = \begin{bmatrix} a(w_2 - w_1) + y_2 y_3 z_1 + x_4(y_2 w_3 + x_3 w_2)\\ b(w_1 + w_2) - y_1 y_3 z_1 - x_4(y_1 w_3 + x_3 w_1)\\ -c w_3 + y_1 y_2 z_1 + x_4(y_2 w_1 + x_1 w_2) \end{bmatrix}, \qquad (7)$$

$$\mathbf{k}(\mathbf{z}, \mathbf{w}) = \begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1 \end{bmatrix}.$$

Theorem: *The error system (5) is minimum phase system.*
Proof:
Choose a storage function candidate:

$$\begin{aligned} V(\mathbf{z}, \mathbf{w}) &= W(\mathbf{z}) + \frac{1}{2}\mathbf{w}^{\mathrm{T}}\mathbf{w},\\ &= W(\mathbf{z}) + \frac{1}{2}\left(w_1^2 + w_2^2 + w_3^2\right), \end{aligned} \qquad (8)$$

where $W(\mathbf{z}) = \frac{1}{2} z_1^2$, $W(\mathbf{0}) = 0$.

The zero dynamics of the system (U23) describes the internal dynamics and occur when $\boldsymbol{w} = \boldsymbol{0}$, i.e.

$$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}) \qquad (9)$$

Differentiating $W(\mathbf{z})$ respect to , we get:

$$\begin{aligned} \dot{W}(\mathbf{z}) &= \frac{\partial W(\mathbf{z})}{\partial \mathbf{z}}\dot{\mathbf{z}} = \frac{\partial W(\mathbf{z})}{\partial \mathbf{z}}\mathbf{f}(\mathbf{z}),\\ &= [z_1][-d z_1],\\ &= -d z_1^2 \le 0. \end{aligned} \qquad (10)$$

Since $W(\mathbf{z}) > 0$ and $\dot{W}(\mathbf{z}) < 0$, it can be concluded that $W(\mathbf{z})$ is the Lyapunov function of $\mathbf{f}(\mathbf{z})$ and $\mathbf{f}(\mathbf{z})$ is globally asymptotically stable which means that the controlled system (5) is *minimum phase* system, and theorem is proved.

Theorem[12-15]: *If the system (6) is a minimum phase system, the system (6) will be equivalent to a passive system and asymptotically stabilized at an equilibrium point if we let the local feedback control as follows:*

$$\mathbf{u} = \mathbf{k}^{-1}(\mathbf{z}, \mathbf{w})\left[-\mathbf{l}(\mathbf{z}, \mathbf{w}) - \left(\frac{\partial W(\mathbf{z})}{\partial \mathbf{z}}\mathbf{g}(\mathbf{z}, \mathbf{w})\right)^T - \varepsilon \mathbf{w} + \xi\right] \qquad (11)$$

where the Lyapunov function of $\boldsymbol{f}(\mathbf{z})$ *is* $W(\mathbf{z})$, ε *is a positive real value and* $\boldsymbol{\xi}$ *is an external signal vector that is connected with the reference input.*

By using (11) we can compute the controller as:

$$\begin{pmatrix} u_1\\ u_2\\ u_3 \end{pmatrix} = \begin{pmatrix} -a(e_2 - e_1) - y_2 y_3 e_4 - x_4(y_2 e_3 + x_3 e_2) - y_2 y_3 e_4 - \varepsilon e_1 + \xi_1\\ -b(e_1 + e_2) + y_1 y_3 e_4 + x_4(y_1 e_3 + x_3 e_1) - x_1 x_3 e_4 - \varepsilon e_2 + \xi_2\\ c e_3 - y_1 y_2 e_4 - x_4(y_2 e_1 + x_1 e_2) - x_1 y_2 e_4 - \varepsilon e_3 + \xi_3 \end{pmatrix} \qquad (12)$$

3. Numerical Simulation

We discuss and illustrate the numerical simulations results between two identical chaotic Qi systems. Systems (2) and (3) with (12) are solved numerically (using e.g. Mathematica 7 software) for $a = 35$, $b = 10$, $c = 1$, $d = 10$, $\varepsilon = 11$, $\xi_1 = \xi_2 = \xi_3 = 0$ and the initial conditions of the drive and the response systems at $t_0 = 0$ are $x_1(0) = 1$, $x_2(0) = 1$, $x_3(0) = 1$, $x_4(0) = 1$ and $y_1(0) = -1$, $y_2(0) = -5$, $y_3(0) = 1$, $y_4(0) = 1$, respectively (the same parameters and initial conditions in[1]). The synchronization of this chaotic attractor is shown in Figure 1, where the oscillations of the drive and response systems rapidly become totally indistinguishable. The synchronization errors plot in the same Figure and demonstrate that synchronization is achieved very fast.

4. Conclusions

This letter makes some comments on the article "Passive and impulsive synchronization of a new four-dimensional chaotic system "[Nonlinear Analysis, doi:10.1016/j.na.2010.09.051]. The error system in[1] is not *minimum phase* system. Thus, the synchronization via passive control is not achieved. So, we offer how to use the passive control method to achieve synchronization of Qi system (2). The synchronization between the state variables of drive and response systems is clear in Fig. 1a, b, c, d. The errors converge to zero after small value of t see Fig. 1e, f, g, h, this shows how our controller is very effective.

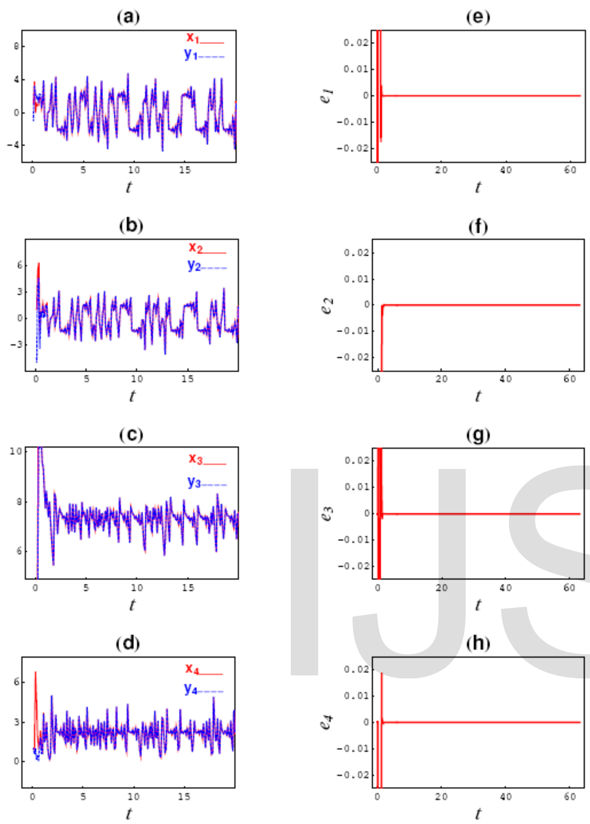

Figure 1. Synchronization of the 4D chaotic system (2) with the passive control method for a=35, b=10, c=1, d=10, $\varepsilon = 11, \xi_1 = \xi_2 = \xi_3 = 0$ and the initial conditions of the drive and the response systems at $t_0 = 0$ are $x_1(0) = 1, x_2(0) = 0, x_3(0) = 1, x_4(0) = 1$ and $y_1(0) = -1, y_2(0) = -5, y_3(0) = 1, y_4(0) = 1$, respectively

REFERENCES

[1] H. Hamiche, K. Kemih, M. Ghanes, G. Zhang, S. Djennoune, Passive and impulsive synchronization of a new four-dimensional chaotic system, Nonlinear Analysis 74 (2011) 1146-1154.

[2] D.Q. Wei, X.S. Luo, B. Zhang, Y.H. Qin, Controlling chaos in space-clamped FitzHugh-Nagumo neuron by adaptive passive method, Nonlinear Analysis: Real World Applications 11 (2010) 1752-1759.

[3] D.Q. Wei, X.S. Luo, Passive adaptive control of chaos in synchronous reluctance motor, Chinese Physics B 17 (2008) 92-97.

[4] K. Kemih, Control of nuclear spin generator system based on passive control, Chaos, Solitons Fractals 41 (2009) 1897-1901.

[5] T. Sangpet, S. Kuntanapreeda, Output feedback control of unified chaotic systems based on feedback passivity, International Journal of Bifurcation and Chaos 20 (2010) 1519-1525.

[6] G.M. Mahmoud, E.E. Mahmoud, A.A. Arafa, Passive control of n-dimensional chaotic complex nonlinear systems, Journal of vibration and control, 19 (2013) 1061–1071.

[7] X.Y. Lou, B.T. Cui, Passivity analysis of integro-differential neural networks with time variable delays, Neurocomputing 70 (2007) 1071-1078.

[8] Q.j. Zhang, J.A. Lu, Passive control and synchronization of hyperchaotic Chen System, Chin Phys B 17 (2008) 492-499.

[9] K. Kemih, Control of nuclear spin generator system based on passive control, Chaos, Solitons Fractals 41 (2009) 1897-1901.

[10] T. Sangpet, S. Kuntanapreeda, Adaptive synchronization of hyperchaotic systems via passivity feedback control with time-varying gains, J Sound Vib. 329 (2010) 2490-2496.

[11] G. Qi, S. Du, G. Chen, Z. Chen, Z. Yuan, On a four-dimensional chaotic system, Chaos, Solitons and Fractals 23 (2005) 1671-1682.

[12] FA-Qiang Wang and C.-X. Liu, Passive control of hyperchaotic lorenz system and circuit experimental on EWB, Int. J. of Modern Phys. B 21 (2007) 3053-3064.

[13] W. Xiang-Jun, L. Jing-Sen, C. Guan-Rong, Chaos synchronization of Rikitake chaotic attractor using the passive control technique, Nonlinear Dyn. 53 (2008) 45-53.

[14] X. Chen, C. Liu, Passive control on a unified chaotic system, Nonlinear Analysis: Real World Applications 11 (2010) 683-687.

[15] Z. Qun-Jiao, L. Jun-An, Passive control and synchronization of hyperchaotic Chen system, Chinese Phys. B 17 (2008) 492-497.